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A scalable architecture for an adiabatic quantum computer has been proposed for superconducting persistent current
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Type II Quantum Computing with Superconductors

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1. Abstract

The results of this research centered on the experimental studies of a single superconducting persistent current qubit, the implementation of type-II algorithms using these qubits, and the proposal for adiabatic quantum computing using these qubits.

The major experimental results on single superconducting persistent current qubits have been the observation of the quantum energy level crossings in niobium qubits, and the microwave measurements of intra-well relaxation times.

We have developed two implementation methods for solving the one-dimensional diffusion equation with a type-II algorithm. In the first method, the state of each qubit is set by the local magnetic field bias. Although this initialization method has the advantage of simplicity, the subsequent unitary collision operations demand precise timing. The second method uses nearly identical qubits that can be addressed locally at the node. Microwave pulses are then used for the initialization, and the unitary transformation is simplified to just timed free propagation.

A scalable architecture for an adiabatic quantum computer has been proposed for superconducting persistent current qubits in which an adiabatically varying magnetic field is applied to all the qubits simultaneously.

2. Summary of Results

The previous work has been focused on three areas: (1) the use of superconducting qubits to implement type-II quantum computing [1-6]; (2) in concert with the DURINT program (the joint AFOSR-ARDA MURI) on type-I quantum computing, the study of single and coupled superconducting persistent current qubits[7,8]; and (3) the proposal of architectures for adiabatic quantum computing [9] using superconducting qubits.

2.1 Persistent Current Qubit (PC- Qubit)

The particular device that we have studied, co-invented by one of us [8, 10], is made from a loop of Nb interrupted by 3 Josephson junctions (Fig. 1). The application of an external magnetic field to the loop induces a circulating current whose magnetic field either adds to (say circulating current in the clockwise direction) or opposes (counterclockwise) the applied magnetic field. When the applied field is near to one-half of a flux quantum, both the clockwise and counterclockwise current states are classically stable. The system behaves as a two-state system. The samples are fabricated at MIT's Lincoln Laboratory in niobium by photolithographic techniques on a trilayer of niobium-aluminum oxide-niobium [11].
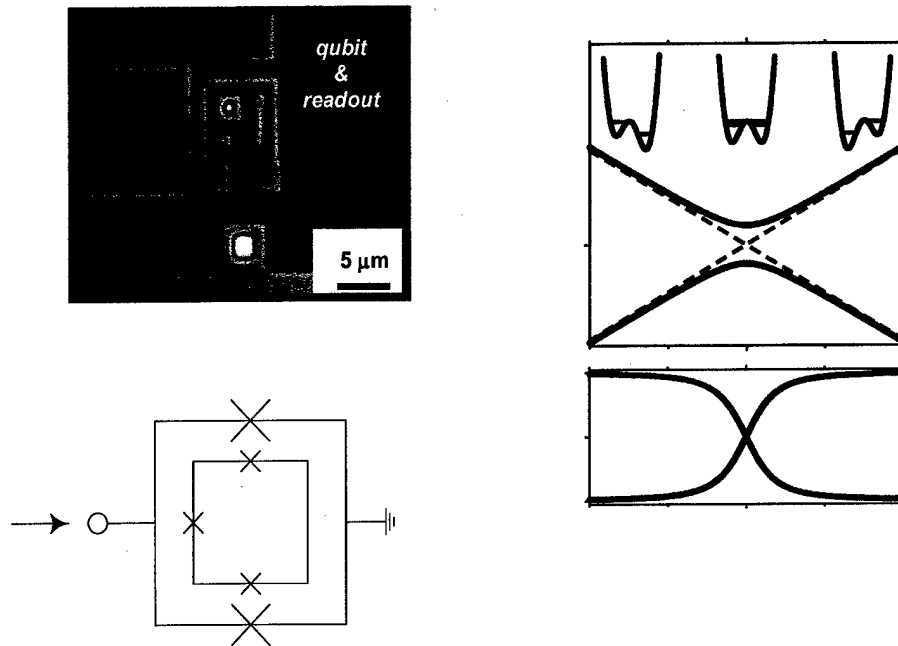
FIGURE 1. (a) SEM image of the persistent current qubit (inner loop) surrounded by the measuring dc SQUID. (b) a schematic of the qubit and measuring SQUID, the x's mark the Josephson junctions. (c) The energy levels for the ground state (dark line) and the first excited state of the qubit versus applied flux. The double well potentials are shown schematically above. The lower graph shows the circulating current in the qubit for both states as a function of applied flux. The units of flux are given in terms of the flux quantum.

The Hamiltonian for the single PC qubit is

$$H_{\text{pcqubit}} = \Phi_0 I_p (f - \frac{1}{2})\sigma_z - V\sigma_x$$

where I_p is the circulating current in the qubit and f is the magnetic flux threading the loop.

Our work on single qubits leverages off our work on type-I quantum computing, and our major experimental results have been

a. Experimental mapping of the potential energy landscape of the persistent current qubit by thermal activation studies[12].

b. Observation of the quantum energy level crossings at 20 mK in niobium qubits fabricated at MIT Lincoln Laboratory.

c. Microwave spectroscopy of the energy levels in the qubits, which agrees with the quantum energy level calculations. Long intra-well relaxation times greater than 50 microseconds were observed, as well as and T_2^* times of about 10 nanoseconds. Both of these times will improve with improved environmental protection.

2.2 Type-II quantum computing

We have developed two implementation methods for solving the one-dimensional diffusion equation with a type-II algorithm:

1. Each type-II algorithm undergoes the following stages: (1) the initialization of the coupled qubits in a node; (2) the collision operator which performs a quantum unitary transformation at the node; (3) the measurement of the qubits; and (4) the streaming of classical information. The hallmark of the first method that we developed is that the state of each qubit is initialized by a static, local magnetic field bias. Although this initialization method has the advantage of simplicity, the subsequent unitary collision operations demand precise timing.

2. The second method maps our superconducting system onto the NMR system that was used to demonstrate the type-II algorithm for the diffusion equation [13]. This method uses nearly identical qubits that can be addressed locally at the node. Microwave pulses are then used for the initialization, and the unitary transformation is simplified to just timed free propagation. Circuits of two coupled qubits have been designed and are being fabricated to study this method.

a. The algorithm

The simplest type II algorithm to implement is the Factorized Quantum Lattice-Gas Algorithm (FQLGA) for the 1D Diffusion Equation [1,4]. It requires a 1D network of classically connected quantum computers (nodes) that consist of only two coupled qubits. Now each node represents a position in a 1D space, with the four quantum states at each spatial position representing a completely unoccupied spatial position ($|0\rangle_a|0\rangle_b$), those

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with a particle moving to the left only ($|1\rangle_a|0\rangle_b$), those with a particle moving to the right only ($|0\rangle_a|1\rangle_b$), and those with particles moving in both directions ($|1\rangle_a|1\rangle_b$). The only collision that can occur between "particles" is for the scenario where the amplitudes of the $|0\rangle_a|1\rangle_b$ and the $|1\rangle_a|0\rangle_b$ states are exchanged. The collision thus simply swaps the middle two particle occupation scenarios. The collision operator that implements this exchange of amplitudes we will call *sqrtswap*, which will be described in more detail below. As for the number density of the fluid, it is just the sum of the probability for each qubit to be in the occupied state.

Note that it is not obvious that the prescription just described should reproduce 1D Diffusion equation. There are lattice requirements and spurious invariant requirements not discussed here which must be satisfied as well.

We will now go through each step in the algorithm in detail.

The first step of the algorithm is the initialization portion, which sets the pre-collision number density $\rho = P_a + P_b$, where P is the probability for a qubit to be in the excited i.e. occupied state:

$$\psi_i = \sqrt{1-P_i}\,|0\rangle + \sqrt{P_i}\,|1\rangle$$

Qubit A represents the occupancy of a particle moving to the left and qubit B represents the occupancy of a particle moving to the right.

Initialize

$\Psi_{1,a}$ $\Psi_{2,a}$ $\Psi_{3,a}$ $\Psi_{4,a}$ $\Psi_{5,a}$ $\Psi_{6,a}$

$\Psi_{1,b}$ $\Psi_{2,b}$ $\Psi_{3,b}$ $\Psi_{4,b}$ $\Psi_{5,b}$ $\Psi_{6,b}$

Collide

Φ_1 Φ_2 Φ_3 Φ_4 Φ_5 Φ_6

Measure

P'_{1a} P'_{2a} P'_{3a} P'_{4a} P'_{5a} P'_{6a}

P'_{1b} P'_{2b} P'_{3b} P'_{4b} P'_{5b} P'_{6b}

Stream*

$P'_{1a} \leftarrow P'_{2a} \leftarrow P'_{3a} \leftarrow P'_{4a} \leftarrow P'_{5a} \leftarrow P'_{6a}$

$P'_{1b} \rightarrow P'_{2b} \rightarrow P'_{3b} \rightarrow P'_{4b} \rightarrow P'_{5b} \rightarrow P'_{6b}$
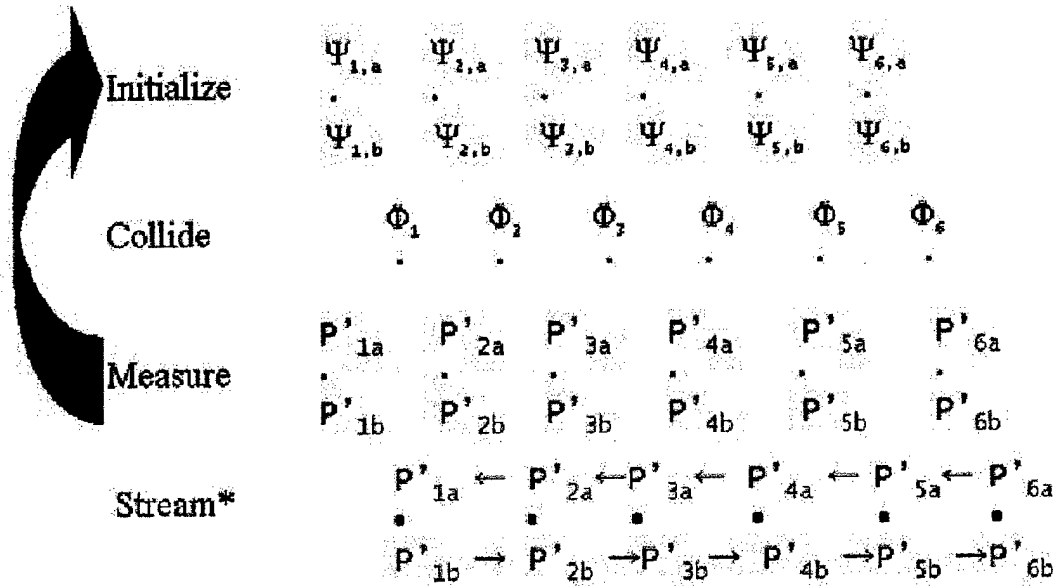
FIGURE 2. Schematic of sequential operations for type-II quantum computation algorithms

The algorithm steps can be visualized as shown above. The first step is to initialize each of the qubits in each node. For simplicity, there are 6 nodes shown, and each node has two qubits. The collision portion of the fluid dynamics is represented by the sqrtswap unitary transformation written as:

$$U = \frac{1}{2}\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1+i & 1-i & 0 \\ 0 & 1-i & 1+i & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

After the collision the system can be represented as being in a single state. The third step is to measure the state of each qubit and then repeat the above three steps to produce a mean occupancy number P' for every qubit. After the calculation of P' is complete, the density of the fluid can be determined by summing the mean occupancies of all the qubits at each node. Lastly, repeat the above for the next time step, initializing the number density with that from the appropriate moving particle from the previous time step. Two methods to implement this series of steps for the 1D Diffusion equation are discussed below.

b. First Implementation Concept

Without considering the common initialization and transformation concepts of quantum computing, this algorithm seems like it could be implemented in a rather simple way. Ignoring all classical necessities like storing the individual measurement results, computing the average of many of these, streaming the occupation numbers, etc., the only quantum mechanical steps are the initialization, the collision, and the measurement.

Let us begin with the simplest step, the measurement. This can be accomplished with an underdamped DC SQUID with our typical ramping/switching measurement technique. The SQUID can distinguish the two states of opposite circulating current due to the opposite fluxes they produce, hence distinguishing σ_z eigenstates.

The initialization portion of the algorithm can be done by taking advantage of the constant σ_x term in our Hamiltonian, which is added to the familiar NMR σ_z term that depends linearly on your tunable external field, here being the flux (f) within the qubit as opposed to the applied magnetic field in NMR. Initialization is then accomplished by letting the qubit relax to its ground state and setting the flux which in turn sets the qubit in any of the continuum of eigenstates that range from $|0\rangle$ to $|1\rangle$ along the Bloch sphere geodesic with real phase.

$$\psi_{ground} = \sqrt{1-P}\,|0\rangle + \sqrt{P}\,|1\rangle$$
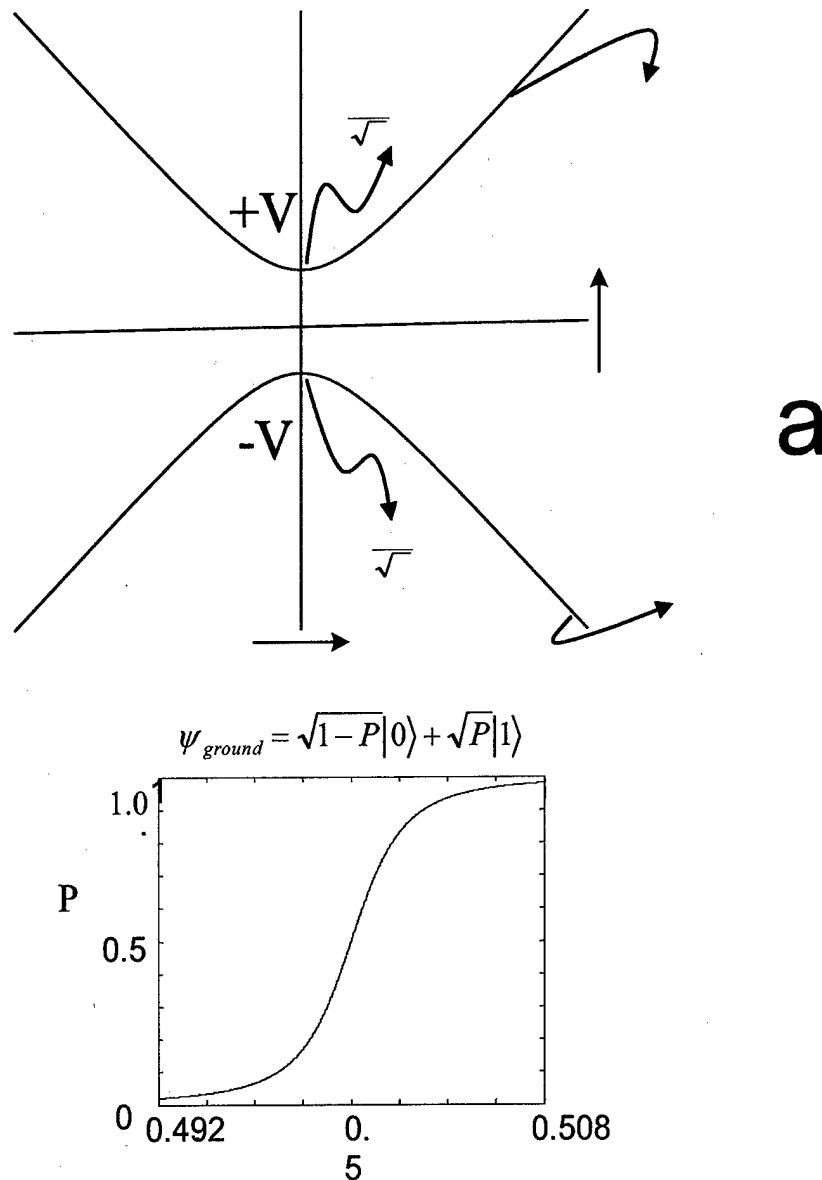
FIGURE 3. (a) Typical avoided crossing diagram showing two bands and the ground and excited states in terms of the asymptote states; (b) Occupancy of ground state as a function of f.

This type of initialization is done while the qubits are uncoupled, which can either be remedied by actually decoupling the qubits during initialization, or by just initializing them while coupled, which would be feasible.

The collision at first glance looks very simple too, but on closer consideration proves to be demanding. The sqrtswap transformation simply swaps the states $|0\rangle_a|1\rangle_b$ and $|1\rangle_a|0\rangle_b$, i.e. performs a Rabi oscillation between these two states such that $\omega_{Rabi} t = \pi$. The first problem arises because our initialization technique has set our coupled Hamiltonian such that the middle two eigenstates in energy are most likely not $|0\rangle_a|1\rangle_b$ and $|1\rangle_a|0\rangle_b$, therefore

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applying radiation at the frequency corresponding to the middle eigenstates' energy splitting for $\omega_{Rabi}t = \pi$ will not perform the desired transformation. One way around this is to find an operating point (f_a, f_b) where the middle two eigenstates are very nearly $|0\rangle_a|1\rangle_b$ and $|1\rangle_a|0\rangle_b$, such that applying radiation at their energy difference for $t = \pi/\omega_{rabi}$ will accomplish the collision. Below is one example of an operating point where the two sets of eigenstates are very similar; there $|2\rangle$ is the first excited state of the coupled system and $|3\rangle$ is the second excited state of the coupled system.

$\langle 01|2\rangle$

$\langle 10|3\rangle$

$f_a(\Phi_0)=.508$

Flux Seen By Qubit B, $f_b(\Phi_0)$
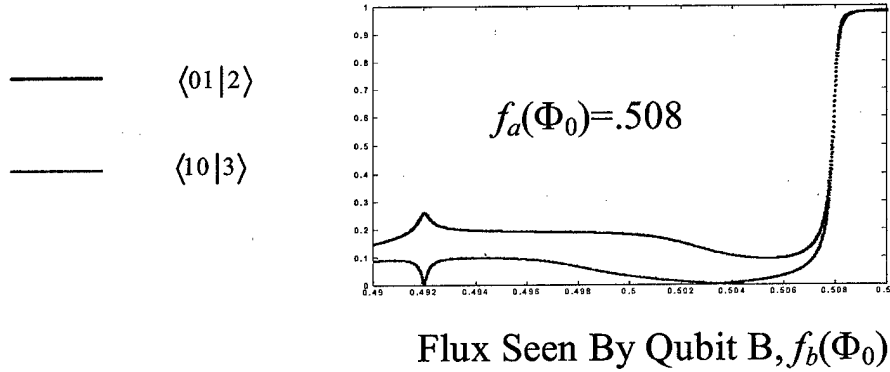
FIGURE 4. Overlap of middle two eigenstates and middle two computational basis states

We have verified both that the energy difference between these two states at this operating point as well as the matrix elements connecting the middle two eigenstates are non-zero, so everything is fine so far. We have also verified that the error introduced from this approximation of the collision does not undermine the results of fluid simulation altogether as can be seen below. We have plotted the number density ρ on the ordinate.
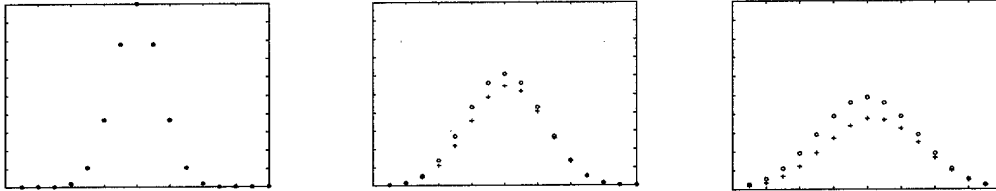
FIGURE 5. Comparison of swap operations and influence on type-II QC

Once the initial states are set (with the qubits uncoupled), the subsequent radiation has to be turned on precisely when you couple the qubits since the qubit states will begin evolving immediately upon coupling, on larmor time scales. *Dealing with larmor time-scale timing is experimentally demanding, making this option very unappealing.*

One could also initialize the qubits while they are coupled, with only a small amount of error, and remove the above constraint, but there is another constraint that appears in

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either initialization technique. To change the operating points from the initializing values to the values that will produce a good collision, one must start the radiation immediately after this is done, on a larmor time scale. To avoid having to control our qubit on this rapid time scale, we have developed an initialization and control method similar to that used in NMR.

c. NMR-like Implementation Concept

Initialization in NMR is typically done by applying radiation at the frequency corresponding to the energy difference between the two single qubit states, with the coupling term ignored as long as the radiation amplitude (expressed as an energy) is sizable compared to the coupling constant. In the rotating frame that is spinning at the radiation frequency around the z-axis, with radiation applied on resonance, the NMR Hamiltonian is:

$$\tilde{\tilde{H}}_{nmr} = \frac{1}{2}\gamma B[\cos(\phi_p)\hat{I}_x + \sin(\phi_p)\hat{I}_y]$$

where ϕ_p is the phase of the radiation, γ is the magnitude of the nuclei's magnetic moment, and B is the magnitude of the applied radio frequency magnetic field.

With this Hamiltonian you can now perform arbitrary single qubit rotations in the rotating frame by applying radiation for times that yield your angle of rotation (θ) via

$$\theta = \frac{1}{2}\gamma Bt$$

using different initial phases to choose the axis of rotation.

To do a sequence of two orthogonal axis rotations for different times to initialize the qubit, one need not have larmor time scale precision. For the three junction persistent current qubit applying radiation on resonance, in the rotating frame spinning at the radiation frequency around the stationary eigenvalue axis z' gives,

$$\tilde{\tilde{H}}_{pcqubit} = \frac{1}{2}g_0\sin(\theta)[\cos(\phi_p)\hat{I}_{x'} + \sin(\phi_p)\hat{I}_{y'}]$$

where g_0 is proportional to the amplitude of your radiation and θ defines your operating point. At $f = \Phi_0/2$,

$$\tilde{\tilde{H}}_{pcqubit}(f = \frac{1}{2}) = \frac{1}{2}g_0[\cos(\phi_p)\hat{I}_z + \sin(\phi_p)\hat{I}_y]$$

where now your rotating frame is spinning around the x-axis.

Measurement of the qubit state must be performed in the σ_x basis so that all computation can be done in the rotating frame at $f = \Phi_0/2$. Operating at $f=\Phi_0/2$ is ideal for low flux noise, but some care must be taken about the measurement. At first glance it

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would seem that the eigenstates are not resolvable with the σ_z measurement, but due to self-inductance effects this is not true.

With the radiation off, in the lab frame, the Hamiltonian for a two qubit NMR system reads:

$$H_{\text{nmr}} = \omega_{o1}\hat{I}_z^1 + \omega_{o2}\hat{I}_z^2 + \frac{2\pi}{\hbar}J_{12}\hat{I}_z^1\hat{I}_z^2$$

Since the measurement is in the σ_z basis, and to preserve computing in the rotating frame, you always go to a rotating frame about the z-axes. In a co-rotating frame at the resonant frequency:

$$\tilde{H}_{\text{nmr}} = \frac{2\pi}{\hbar}J_{12}\hat{I}_z^1\hat{I}_z^2$$

From here it is straightforward to get to any other imaginable two qubit operation. For the persistent current qubit things are a bit more complicated since the coupling term is not in the same direction as the single qubit terms at $f = \Phi_0/2$:

$$H_{\text{pcqubit}} = \omega_{o1}\hat{I}_x^1 + \omega_{o2}\hat{I}_x^2 + \frac{2\pi}{\hbar}J_{12}\hat{I}_z^1\hat{I}_z^2$$

Preserving the ability to compute in the rotating frame forces one to rotate both qubits about the x-axis at $f = \Phi_0/2$, making the transformation less trivial than in the NMR case, resulting in:

$$\tilde{H}_{\text{pcqubit}} = \frac{\pi}{\hbar}J_{12}[\hat{I}_z^1\hat{I}_z^2 + \hat{I}_y^1\hat{I}_y^2]$$

where the constraint that $\omega_1 = \omega_2$ has been imposed. This removes the ability to address the qubits separately via frequency discrimination, and forces one to address them separately spatially, i.e. each seeing a different radiation source. Fortunately, one benefit of solid state computing is that each qubit can have its own personal antenna.

Now as to the exact collision operator we need to implement, we have the following:

$$sqrtswap = \frac{1}{2}\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1+i & 1-i & 0 \\ 0 & 1-i & 1+i & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} = \exp[-i\frac{\pi}{8}(\sigma_z^1\sigma_z^2 + \sigma_y^1\sigma_y^2 + \sigma_x^1\sigma_x^2)]$$

Since the arguments in the exponential commute you can write the collision matrix as:

$$sqrtswap = \exp[-i\frac{\pi}{8}(\sigma_z^1\sigma_z^2 + \sigma_y^1\sigma_y^2)]\exp[-i\frac{\pi}{8}\sigma_x^1\sigma_x^2]$$

The first term is just free evolution in the doubly rotating frame. The second term can be written as:

$$\exp[-i\frac{\pi}{8}\sigma_x^1\sigma_x^2] = R_y^1(\frac{\pi}{2})R_y^2(\frac{\pi}{2})\exp[-i\frac{\pi}{8}\sigma_z^1\sigma_z^2]R_y^1(-\frac{\pi}{2})R_y^2(-\frac{\pi}{2})$$

The middle term on the right hand side can then be achieved in the rotating frame by

$$\exp[-i\frac{\pi}{8}\sigma_z^1\sigma_z^2] = \exp[-i\frac{\pi}{8}(\sigma_z^1\sigma_z^2 + \sigma_y^1\sigma_y^2)]R_Z^1(\pi)\exp[-i\frac{\pi}{8}(\sigma_z^1\sigma_z^2 + \sigma_y^1\sigma_y^2)]R_Z^1(\pi)$$

And so at the price of locally addressing each qubit with radiation, which was done in the first scheme except with a static field for initialization, now only pulses on the Rabi time scale are needed, and none on the larmor scale.

In summary, the following design criteria must be met for the second implementation scheme:

1. The two qubits must be identical so that your coupled free evolution is static in the doubly rotating frame.

2. The radiation must be applied such that it is mostly seen by only the desired qubit, one radiation line for each qubit.

3. The radiation must be at least 50 times greater in magnitude than the coupling constant, so that the coupled static term in the rotating frame can be ignored during initialization and any other single qubit operations.

4. The DC SQUID must have no resonances at the energy splitting of the qubits.

2.3. Adiabatic quantum computing

A scalable architecture for an adiabatic quantum computer has been proposed for superconducting persistent current qubits in which the computation is performed by slowly varying a magnetic field applied to all the qubits simultaneously[14]. Only local incoherent operations are needed, such as switching on and off the pairwise interaction, and measurement of only a small subset of the qubits.

Adiabatic quantum computation [9] is a recently proposed, general approach to solving computational problems of the complexity class NP via energy minimization. In particular, by exploiting the ability of coherent quantum systems to follow adiabatically the ground state of a slowly changing Hamiltonian, it aims to bypass automatically the many separated local minima that occur in difficult minimization problems and confound all known classical heuristics. Adiabatic quantum computation is of theoretical interest because it provides a straightforward, non-oracular way to pose class NP problems on a quantum computer, and most research on it to date focuses on ascertaining its time complexity [9, 15, 16]. However, it is also of potentially great practical interest because encoding a quantum computation in a single eigenstate, the ground state, offers intrinsic protection against dephasing and dissipation [17].

We also discuss the implementation scheme of Kaminsky, Lloyd, and Orlando for using superconducting persistent-flux qubit. This proposed architecture is robust in the face of manufacturing imperfections, decoherence, and noise. They found the following criteria for adiabatic QC architecture:

1. A starting Hamiltonian with an easily initialized ground state.

2. Initial and problem Hamiltonians that share no symmetries so as to ensure levels do not cross.

3. To minimize unwanted components to the interqubit couplings

4. To equalize the mutual inductances between qubits to be coupled, even if they are not nearest neighbors

5. To allow room for the shrinkage of the energy gap expected when interpolating between Hamiltonians.

To have a starting Hamiltonian with an easily initialized ground state, initialize the qubits with f = 1/2 flux quanta so that each qubit's σ_z term dominates all other terms affecting the qubit. This choice allows the interqubit couplings to be *switched on for the entire computation.* Switching couplings on and off is needed only to program the computer; it is not necessary during any computation.

To have the initial and problem Hamiltonians that share no symmetries so as to ensure levels do not cross, use a 4-junction, rather than 3-junction PC-qubit. The four-junction qubit replaces the qubit with the lower critical current with a tunable parallel junction, so as to allow XX and XZ inductive couplings as well as ZZ.

To minimize unwanted components to the interqubit couplings, use $\sigma_z + \sigma_x$ as basis of the problem Hamiltonian. This naturally incorporates the inevitable mixed $\sigma_z \sigma_x$ terms when 4-junction qubits are inductively coupled simply by having the coupling loop arranged symmetrically so as to have equal mutual inductances to both loops of the 4-junction qubit, since:

$$(\sigma_{Z1} + \sigma_{X1})(\sigma_{Z2} + \sigma_{X2}) = \sigma_{Z1}\sigma_{Z2} + \sigma_{Z1}\sigma_{X2} + \sigma_{X1}\sigma_{Z2} + \sigma_{X1}\sigma_{X2}$$
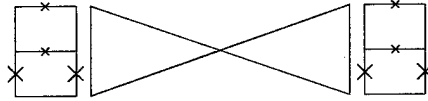

FIGURE 5. schematic: not to scale, inductive loop much longer than qubit dimensions

To equalize the mutual inductances between qubits to be coupled, even if they are not nearest neighbors, it is essential that the buses through which non-nearest neighbors are coupled have as low self-inductances as possible. This criteria follows from the fact that the qubit-qubit mutual inductance via a coupling loop goes ashe ratio of the square of the mutual inductance of the qubit-coupling loop to the self inductance of the coupling loop.A combination of minimizing the loop area of the buses, twisting their wires, and/or placing ground planes beneath them should accomplish this.

Lastly, to allow room for the shrinkage of the energy gap expected when interpolating between Hamiltonians, Optimize junction parameters so as to maximize tunnel splitting and critical current (so as to maximize MI_c^2) . For example, numerical calculations on the full 4-junction qubit Hamiltonian for our current technology of Nb junctions of $J_C = 2$ $\mu A/\mu m^2$ of sizes $(0.85\ \mu m)^2$ and $(0.65\ \mu m)^2$ yields:

Energy Difference between Qubit Levels = 3.7 GHz (190 mK)

Inductive Coupling >> 4.5 GHz/pH (225 mK/pH)

Thus, given the apparent $O(n^{-1})$ behavior of gap, current technology should permit demonstrations of the ideal adiabatic algorithm with several qubits and of the tunneling-based heuristic with many more.

Future Josephson junction technology could provide Josephson energies 10-100 times higher than those currently used in our PC qubits and thus allow for ideal adiabatic computers with 10's or 100's of qubits, and quantum annealers with 1000's of qubits.

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5. Personnel Supported

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6. Publications

1. *T.P. Orlando, S. Lloyd, L.S. Levitov, K.K. Berggren, M.J. Feldman, M.F. Bocko, J.E. Mooij, C.J.P. Harmans, C.H. van der Wal, "Flux-Based Superconducting Qubits for Quantum Computation," *Physica C-Superconductivity and Its Applications,* vol. 372, pp.194-200, 2002.

2. *T. P. Orlando, Lin Tian, D. S. Crankshaw, S. Lloyd, C. H. van der Wal, J. E. Mooij and F. Wilhelm, "Engineering the quantum measurement process for the persistent current qubit," *Physica C: Superconductivity, Volume 368, Issues 1-4, 1 March 2002, Pages 294-299*

3. *William M. Kaminsky and Seth Lloyd, "Scalable Architecture for Adiabatic Quantum Computing of NP-hard problems, in Quantum computing and Quantum Bits in Mesoscopic Systems, Kluwer Academic, 2003.

4. D. Nakada, K.K. Berggren, E. Macedo, V. Liberman, T.P. Orlando, "Improved Critical-Current-Density Uniformity by Using Anodization," IEEE Transaction of Applied Superconductivity, vol. 13, no. 2, pp. 111-4, June 2003.

5. K. Segall, D.S. Crankshaw, D. Nakada, T.P. Orlando, L.S. Levitov, S. Lloyd, M. Tinkham, N. Markovic, S. Valenzuela, and K.K. Berggren, "Impact of Time-Ordered Measurements of the Two States in a Niobium Superconducting Qubit Structure," Phys. Rev. B Rapid Communications, vol. 67, pp. 220506-1-4, 2003.

6. K. Segall, D.S. Crankshaw, D. Nakada, B. Singh, J. Lee, T.P. Orlando, K.K. Berggren, N. Markovic, S.O. Valenzuela, and M. Tinkham, "Experimental Characterization of the Two Current States in a Nb Persistent-Current Qubit," IEEE Transaction of Applied Superconductivity., vol. 13, no. 2, pp.1009-12, June 2003.

7. D.S. Crankshaw, K. Segall, D. Nakada, T.P. Orlando, L.S. Levitov, S. Lloyd, S.O. Valenzuela, N. Markovic, M. Tinkham, K.K. Berggren, "DC Spectroscopy of Macroscopic Quantum Levels in a Superconducting Qubit Structure with a Time-Ordered Meter," submitted for publication.